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Half-branes, singular brane intersections, and Kaluza-Klein reduction

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Abstract: Two subtle aspects of brane intersections are investigated. The first concerns the 'half-branes' that arise in discussions of the Hanany-Witten effect, often in the D0/D8 setting. The second involves the validity of seemingly singular classical BPS brane intersections. A study of holomorphic curves in the background of a Kaluza-Klein monopole and the associated reduction to type-IIA supergravity sheds light on both issues. Many seemingly singular D2/D6 intersections are shown to lift to smooth configurations of M2-branes in 11-dimensions, and a mechanism is found for certain $\mathbb{Z}_2$ confinement effects in type II string theories that eliminates any need for half-branes.

Keywords: M-Theory, p-branes, D-branes.

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Contents

1. Introduction

In recent years the study of intersecting branes, especially of the BPS sort, has provided fertile ground for the development of ideas in string theory, gauge theory, and gravity. For example, BPS intersecting brane scenarios provided the setting for the accounting of black hole entropy in the case studied by Strominger and Vafa [1] and for the subsequent generalizations. In addition, the Hanany-Witten framework [2] and the Maldacena conjecture [3] have forged strong links between gauge theories and brane intersections. As a result, the construction and study of intersecting brane solutions has become a minor industry.

The case of intersections with D6-branes is particularly tractable due to the fact that a unit charged D6-brane in ten dimensions lifts to the Kaluza-Klein monopole of 11-dimensional supergravity. This solution is just the usual $4+1$ Kaluza-Klein monopole [4] times a flat 6-dimensional space. The important property is that the unit charged monopole solution is completely smooth, so that it can be approximated by flat Minkowski space at its center. More generally, the 'core' of the multi-charged Kaluza-Klein monopole is just an orbifold singularity. This means that the 'near-core' versions of certain ten-dimensional supergravity solutions involving intersections with D6-branes can be constructed [5, 6, 7] by an appropriate quotient of branes in 11-dimensional Minkowski space. We note that for appropriately chosen parameters (large internal S^1), the curvature of a unit charged Kaluza-Klein monopole is everywhere small, so that a description in terms of classical supergravity is appropriate.

When a full supergravity solution is not available, one can often extract useful information by considering one brane as a 'test' or 'probe' brane in a background spacetime determined by another brane (see, e.g., [8]–[12] and others in the intersecting brane context). This description is appropriate when the test brane is much lighter than the background brane. When derivatives on the brane are appropriately small (see, [13] for a review), static configurations of the test branes are determined by the Dirac-Born-Infeld action. In the BPS case, the Dirac-Born-Infeld action may lead to BPS solutions even in the presence of large curvatures [14].

In this context, D6-branes again provide a particularly tractable setting. For example, a test D2-brane in the background of a D6-brane is described in M-theory as a test M2-brane in the background of a Kaluza-Klein monopole. The point here is that the Kaluza-Klein manifold is Kähler (in fact, hyper-Kähler). In a static spacetime of the form $K \times R$ where K is Kähler and R is the time direction, static BPS configurations of M2-branes are exactly described by holomorphic curves in K. This simplifying property was used in [10, 11, 12] to study the Hanany-Witten brane-creation effect.

Here, we continue the study of test D2-branes intersecting D6-branes by mapping out in detail the associated charge distributions in ten-dimensions. Our goal here is two-fold. First, we wish to investigate the issue of half-branes that arises naturally in discussions of the Hanany-Witten effect, e.g. [2][15]–[26]. Consider for example a system with a single D0-brane and a single D8-brane and suppose that the boundary conditions are such that the ten-form Ramond-Ramond gauge field takes the symmetric values $\pm 1/2$ of the fundamental quantum on either side of the D8-brane domain wall. Then, certain arguments [15, 19] (and [16, 17, 18, 20, 22, 21, 27] in related contexts) in massive type-IIA supergravity [28] lead to the conclusion that exactly 1/2 of a fundamental string must end on the D0-brane. While this seems to be at odds with charge quantization, several possible resolutions immediately present themselves. One possibility is that the half-string is a mere artifact of some accounting scheme (see, e.g. [29]–[33]) and that it is not in fact in conflict with charge quantization. Another possibility is that such symmetric boundary conditions for D8-branes are not in fact allowed, and that the Ramond-Ramond gauge ten-form field strength F_{10} must in fact take integer values. A final possibility is that F_{10} is in fact allowed to take half-integer values but that, in such backgrounds, D0-brane charge is allowed to occur only in multiples of 2. By considering the T-dual D2/D6 system, we will uncover evidence in support of this final alternative of $\mathbb{Z}_2$ confinement of D0-brane charge.

Our second goal is to investigate in detail the case where the branes actually meet and intersect. The point here is that the background metric and/or the dilaton is typically singular at the location of the background brane. Thus, a priori, the Dirac-Born-Infeld effective action is not an adequate description of the test brane at the point where it actually intersects the background brane.[1] However, as mentioned

[1]In the particular case of the Neveu-Schwarz 5-brane there is an exact conformal field theory in

above, the corresponding core region is smooth for the unit charge Kaluza-Klein monopole. Thus, studying smooth test M2-branes in 11-dimensions should tell us about the allowed intersections of D2- and D6-branes in ten dimensions. As we will see, some of these intersections look quite singular from the ten-dimensional point of view. In particular, we find that many solutions analogous to the fractional string solutions of [9] correspond to smooth M2-branes in 11-dimensions. Nevertheless, the proper string charge always appears in integral quanta.

The plan of our paper is as follows. In section 2, we consider a holomorphic curve in the Kaluza-Klein monopole background which has been previously discussed [12] in connection with the Hanany-Witten effect and compute its reduction to the type-IIA D6-background. Although the corresponding D2-brane does not in fact intersect the D6-brane, it illustrates a number of effects and provides some insight into the question of half-branes. The associated charge distributions are described in terms of the three notions of charge reviewed in [32]: brane source charge, Maxwell charge, and Page charge [36]. This will show that certain fractional brane issues in the D2/D6-context are resolved (as in the D0/D2-brane case of [29, 30, 31, 33], see also [32]) by proper accounting techniques. However, we will also see that, in this particular example, the system cannot be compactified along the D2-brane. As a result, T-duality makes no connections with the case of the D0/D8-system.

Section 3 then considers holomorphic curves for which the corresponding D2-branes do intersect the D6-brane. In fact, we consider the most general smooth holomorphic curves whose reduction to type IIA yields rotationally symmetric D2-branes. One particular class of such D2-branes has an analogue in the compactified system. Curves in this class feature an essentially flat D2-brane connected to the D6-brane by a thin tube that approximates a collection of fundamental strings in much the same way as the BIons of [37]. The fundamental string Page charge of this tube is one unit. However, the D2-brane itself necessarily has two units of D2-brane charge. Multiple holomorphic curves lead to an integer number of fundamental strings and an even number of D2-branes. Thus, the T-dual D0/D8 system necessarily contains an even number of D0-branes. This argues for the $\mathbb{Z}_2$ confinement effect described above. We close with a discussion of various issues in section 4, including extensions to non-BPS configurations and a comparison with similar $\mathbb{Z}_2$ confinement effects for five-branes in type-I string theory [38, 39].

2. Half-branes and the reduction of holomorphic curves

We consider below the holomorphic curves of [12] in the Kaluza-Klein monopole background and reduce them to configurations of test D2-branes and fundamental strings in the background of a unit charged D6-brane. Supposing that the D6-brane

this region with which one can work, see e.g. [34, 35].

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is oriented along the $x_0, x_1, x_2, \ldots, x_6$ directions and introducing $dx_{\|}^2 = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2 + dx_6^2$ and $dx_{\perp}^2 = dx_7^2 + dx_8^2 + dx_9^2$, the type-IIA background fields take the form

$$\begin{aligned} ds^2_{\text{string}} &= V^{-1/2} dx_{\|}^2 + V^{1/2} dx_{\perp}^2 \\ e^{2\phi} &= V^{-3/2} \\ A_1 &= \frac{1}{2}(1-\cos\theta) d\psi \\ F_2 &= \frac{1}{2} \sin\theta d\theta \wedge d\psi \,, \end{aligned} \tag{2.1}$$

where $V = 1 + 1/2r$, $r = x_7^2 + x_8^2 + x_9^2$, $\theta = \cos^{-1}(-x_9/r)$, and $\psi = \tan^{-1}(x_8/x_7)$. Here, to simplify the formulas we have set the radius R_{10} of the M-theory circle to one.

We will be interested in solutions in which the D2-brane is extended in the x_7, x_8 directions. As a result, one can visualize this system by suppressing the dimensions along the D6-brane and drawing only the three space x_7, x_8, x_9, in which the D6-brane appears as a point object. Here, we consider the case where the branes do not intersect, but instead the D2-brane will pass above or below the D6-brane. This is analogous to the $\psi_0 = \pi$ case of [35]. The intersecting case will be investigated in section 3.

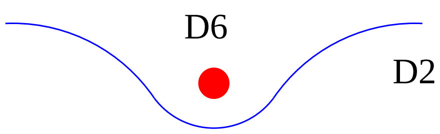

Figure 1: A sketch of our brane configuration.

2.1 The brane source charge

It is convenient to describe the D2 brane by the electric current it carries. When Chern-Simons terms are relevant, there are several different definitions of a charge current, each of which is useful in certain settings. For definiteness, we consider the M2 'brane-source' current j^{bs}_{M2} of [32], which is the current that arises by varying the action with respect to the three-form gauge field A_3^M of eleven-dimensional supergravity and multiplying by $2\kappa_{11}^2$, the gravitational coupling in eleven dimensions. Our normalizations are fixed by stating that the bosonic part of the eleven-dimensional supergravity action is

$$S_{\text{bosonic}} = \frac{1}{2\kappa_{11}^2} \int d^{11}x (-g)^{1/2} \left(R - \frac{1}{2}|F_4^M|^2 \right) - \frac{1}{6} \int A_3^M \wedge F_4^M \wedge F_4^M \,, \tag{2.2}$$

where $F_4^M = dA_3^M$. Note that, when the Chern-Simons term above can be neglected, the equation of motion for A_3^M in the presence of a source is

$$d * F_4^M = * j^{bs}_{M2} \,. \tag{2.3}$$

One can check that, since no M5-branes are present, the supergravity equations of motion imply conservation of this current $d * j^{bs}_{M2} = 0$, and that the brane-source current agrees with the Maxwell and Page currents [32].

We will be interested in calculating the D2-brane and fundamental string (F1) brane-source currents resulting from the Kaluza-Klein reduction to ten dimensions. Thus we consider eleven-dimensional solutions with a spacelike Killing vector field λ^α, normalized so that $|\lambda| = 1$ at infinity. Again the F1 and D2 brane-source currents in type IIA are given by varying the action with respect to gauge fields, in this case the Neveu-Schwarz two-form B_2 and the Ramond-Ramond 3-form A_3. A useful way to write the relation between these currents and j_{M2} is in terms of the one-form $\tilde{\lambda}$ constructed from the Killing field λ^α by lowering the index and renormalizing to set $\tilde{\lambda}_\alpha \lambda^\alpha = 1$; i.e., we set $\tilde{\lambda}_\alpha = \lambda_\alpha/|\lambda|^2$. Such a one form can be expressed in terms of a coordinate x_{10} on the Killing orbits and the associated Kaluza-Klein gauge field A_1 as $\tilde{\lambda} = dx_{10} + A_1$. One can then verify from the supergravity equations of motion that the currents are related by

$$*j^{bs}_{M2} = *_s j^{bs}_{D2} \wedge \tilde{\lambda} - *_s j^{bs}_{F1}\,, \tag{2.4}$$

or

$$j^{bs}_{M2} = e^{2\phi/3} j^{bs}_{D2} - e^{8\phi/3} j^{bs}_{F1} \wedge \tilde{\lambda}\,. \tag{2.5}$$

Here, the star $(*)$ denotes the Hodge dual with respect to the metric of 11-dimensional supergravity, while the star $(*_s)$ with an s subscript denotes the Hodge dual with respect to the type-IIA string metric. The usual type-IIA dilaton is represented by ϕ. The ten-dimensional spacetime $\mathcal{M}_{10}$ is the quotient of the 11-dimensional spacetime $\mathcal{M}_{11}$ by the S^1 orbits of λ^α and the type-IIA currents have been pulled back to the 11-dimensional spacetime via this quotient map. Equations (2.4) and (2.5) is one precise version of the statement that fundamental strings are associated with that part of the M2-brane current that flows around the x_{10} direction.

A useful form of the metric for the 11-dimensional Kaluza-Klein monopole is [10, 11, 12]

$$ds^2 = -dx_{\|}^2 + V dv d\overline{v} + V^{-1} \left| \frac{dw}{w} - f dv \right|^2 , \tag{2.6}$$

where

$$f = \frac{x_9 + r}{2vr}\,, \tag{2.7}$$

correcting a small typographic error in [10, 12]. The complex coordinates v and w define one of the complex structures on the euclidean Taub-Nut space. They are related to the ten-dimensional coordinates through

$$\begin{aligned} v &= x_7 + i x_8 \\ w &= e^{-(x_9 + i x_{10})} \left(-x_9 + \sqrt{x_9^2 + |v|^2} \right)^{1/2}, \end{aligned} \tag{2.8}$$

and Kaluza-Klein reduction takes place along the Killing field $\partial_{x_{10}}$. Such coordinates are smooth so long as $v \neq 0$ or $x_9 < 0$. Note that x_{10} ranges over $[0, 2\pi]$ consistent with our setting $R_{10} = 1$.

Reference [12] in fact describes two interesting holomorphic curves given by $w = e^{-b}$ and $w = e^{-b}v$ for some complex constant b. A symmetry of the form $(w, v) \to (v/w, v)$ interchanges these two families of curves, so that we need only consider one of them. A useful observation is that this symmetry changes the sign of x_9, but leaves r invariant. When $b + \bar{b}$ is large and negative, the first curve describes a mostly flat M2-brane located at large negative x_9 oriented along the x_7, x_8 directions. For large positive $b + \bar{b}$, this curve is tightly cupped around the monopole.

Let us consider the first curve, $w = e^{-b}$, in detail. One can check that this curve has $x_9 < 0$ for $x_7 = x_8 = 0$ and so is manifestly smooth. Some calculation shows that an M2-brane lying on such a curve at $x_1, x_2, x_3, x_4, x_5, x_6 = 0$ is associated with the current

$$j^{bs}_{M2} = \frac{i2\kappa_{11}^2 T_{M2}}{2} \delta^{(6)}(x) \delta^{(2)}(w - e^{-b}) \frac{w\overline{w}}{V^2} dx_0 \wedge \omega \wedge \overline{\omega}\,, \tag{2.9}$$

where

$$\omega = (V^2 + f\overline{f})dv - \overline{f}\frac{dw}{w}\,, \tag{2.10}$$

and T_{M2} is the tension of the M2-brane.

To pick out the part of j^{bs}_{M2} associated with fundamental strings, we see from (2.5) that one need only contract j^{bs}_{M2} (on the final index) with the Killing field $\lambda = \partial_{x_{10}}$. At this stage, it is useful to introduce the cylindrical radial coordinate $\rho = |v|$. The result takes the form

$$j^{bs}_{M2} \cdot \lambda = -\frac{2\kappa_{11}^2 T_{M2}}{2} \delta^{(6)}(x) \delta^{(2)}(w - e^{-b}) \frac{\tilde{f} w\overline{w}}{\rho} dx_0 \wedge \left(d\rho + \frac{\tilde{f}}{2V\rho} dx_9 \right), \tag{2.11}$$

where we have introduced $\tilde{f} = 1 - \cos\theta$ so that the type-IIA magnetic 1-form potential is $A_1 = \frac{1}{2}\tilde{f} d\psi$. One sees that (2.11) does not project directly to the ten-dimensional spacetime as the delta functions depend on x_{10}. This is merely a reflection of the fact that translations along the Killing field λ do not map our holomorphic curve onto itself. Nevertheless, at events that are far from the curve as compared with the size of the S^1 orbits, type-IIA supergravity should still be a good description of the spacetime obtained by placing an M2-brane on this curve. In this regime, the type-IIA fields are related to the average values of the current over the S^1. Let us therefore average the current (2.11) over the orbits of the Killing field by multiplying by $1/2\pi$ and integrating over dx_{10}. Multiplying (2.11) by the appropriate dilaton factors and adjusting the sign as dictated by (2.5) yields the associated fundamental string current. In the curved spacetime context, it is the dual current $*j_{F1}$ which is most easily interpreted. Here, $*$ represents the Hodge dual defined by the string metric of the D6-brane spacetime. The result may be written

$$*j_{F1} = \frac{\tilde{f} 2\kappa_{10} T_{F1}}{2} \delta^{(6)}(x) \wedge_{i=1}^{6} dx_i \wedge \left(\delta(x_9 - \hat{x}_9(\rho)) \frac{d\psi}{2\pi} \wedge dx_9 + \delta(\rho - \hat{\rho}(x_9)) d\rho \wedge \frac{d\psi}{2\pi} \right), \tag{2.12}$$

where $\hat{x}_9(\rho)$ and $\hat{\rho}(x_9)$ are the functions determined by the relation $|w| = |e^{-b}|$, the factor T_{F1} is the tension of a fundamental string, and $2\kappa_{10}$ is the ten-dimensional gravitational coupling. In arriving at (2.12) we have used the relation $2\kappa_{11}^2 T_{M2} = 2\kappa_{10}^2 T_{F1}$.

Note that, due to the smearing, the fundamental string current j_{F1} in fact has support on a submanifold with two spatial dimensions. This submanifold is just the worldsheet of the D2-brane obtained by dimensional reduction of the holomorphic curve. Thus, the strings are dissolved in the D2-brane, and the string charge runs outward along the D2-brane in the radial direction. Note in particular that this configuration is rotationally symmetric about the x_9 axis. The resulting shape of the D2-brane is quite similar to that of [8, 9, 7].

In the above form, the number of fundamental strings passing through a hypersurface $x_0 =$ const. with either $\rho =$ const. or $x_9 =$ const. is readily seen to be $\tilde{f}/2 = \frac{1-\cos\theta}{2}$, where θ is determined by the intersection of the hypersurface with the worldsheet of the M2-brane (or, equivalently, of the resulting D2-brane). The fundamental string current vanishes on the x_9-axis, but represents half a string exiting to infinity (where $\theta = \pi/2$ due to the fact that the D2-brane becomes very flat).

It is instructive to note what occurs when the parameter b is adjusted so that the D2-brane appears to move far past the D6-brane. In this case, the surface of the D2-brane deforms as shown below. One can see that, at intermediate values of x_9, the surface takes the shape of a long thin tube; i.e., a string. In such a configuration, the gauge field strength $\tilde{F}_{[4]}$ that couples to D2-branes is nearly zero, as a cylinder has no net D2-brane charge. Instead, the string charge will dominate the coupling to the supergravity fields.[2] On this string, θ is essentially zero so that the string carries a full unit of fundamental string brane-source charge.

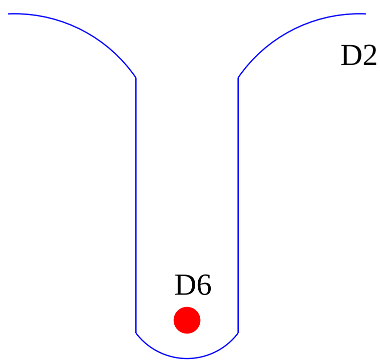

Figure 2: A D2-brane is moved far past a D6-brane.

To understand the general features of the above configurations, we recall from [32] that the brane source currents are not in general conserved. In particular, taking an exterior derivative of (2.4) and using conservation of $*j_{M2}^{bs}$ (which holds in the present context free of M5-branes) yields the relation

$$d * j_{F1}^{bs} = - * j_{D2}^{bs} \wedge F_2 \,. \tag{2.13}$$

[2] Some of the details can be found in [40, appendix] for the more familiar BIons of [37].

We refer to this equation as a constraint as it has components that involve no time derivatives. Here, $F_2 = dA_1$ represents the magnetic flux from the D6-brane. The normalization is such that, as our D2 brane will capture half of the flux from the D6-brane, it is in fact a net source of one-half unit of brane source charge. To see this, note that the total flux (2.1) from the D6-brane is $2\pi R_{10} = 2\pi$, which matches the ratio T_{F1}/T_{D2}. An analogous discussion from the point of view of the worldvolume theory can be found in [18, 20]. Such string charge is created in our solution though the resulting 'half-string' does not leave the D2-brane. Instead, it flows out along the D2-brane to infinity. The $\tilde{f}$ factor in (2.12) describes the gradual creation of this fundamental string current as F_2 flux is captured by the D2-brane. From this and the normalization of (2.13) we can verify that, as expected, the associated j^{bs}_{D2} represents exactly one D2-brane lying on the surface $\rho = \hat{\rho}(x_9)$.

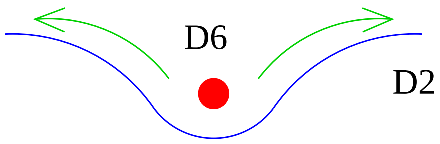

Figure 3: The D2-brane carries F1 charge radially outward.

One might ask if the D6-brane should also be the source of a string. Indeed, there is a constraint in type-IIA theory which states that a D6-brane should be the source of fundamental string brane-source current when it captures an electric flux of the gauge field $\tilde{F}_4$ associated with a D2-brane. Here, the relevant quantity is the 'improved field strength' $\tilde{F}_4 = dA_3 - A_1 \wedge H_3$, which is the fully gauge invariant quantity. The electric flux is computed in terms of the dual $*_s\tilde{F}_4$. Now, since we have treated the D2-brane as a test object, the associated A_3 gauge flux is not readily apparent. Nevertheless, a short argument shows that the associated field strength $*\tilde{F}_4$ must in fact vanish at the D6-brane. The point is that this field strength is related to the 4-form F_4^M of 11-dimensional supergravity through

$$*F_4^M = -e^{-2\phi} *_s H_3 + *_s\tilde{F}_4 \wedge \tilde{\lambda}\,. \tag{2.14}$$

Now, since the M2-brane does not intersect the core of the Kaluza-Klein monopole, $*F_4^M$ should be smooth there. Since λ is a smooth vector field, the contraction $*F_4^M \cdot \lambda$ must be smooth and must vanish whenever λ vanishes, such as at the core of the monopole. Since this contraction is just $*\tilde{F}_4$, we see that see that this flux vanishes at the D6-brane.

2.2 The Maxwell and Page charges

Let us now look at the Maxwell current defined [32] by $d\left(e^{-2\phi} *_s H_3\right) = *_s j^{\text{Maxwell}}_{F1}$. This may be written in terms of the brane source current as

$$*_s j^{\text{Maxwell}}_{F1} = *_s j_{F1} + *_s\tilde{F}_4 \wedge F_2 \tag{2.15}$$

Maxwell charge is always 'diffuse,' in the sense that it is carried by the bulk fields and so is not localized in branes. Some useful insight into the distribution of Maxwell charge in this system can be obtained by integrating the second (diffuse) term over conveniently chosen volumes. In particular, suppose that we integrate this term over an 8-surface V_8 defined by $\rho = \rho_0$ for $x_9^{\min} < x_9 < x_9^{\max}$ and $\psi^{\min} < \psi < \psi^{\max}$. The coordinates $x_1, \ldots, x_6$ are allowed to run from $-\infty$ to ∞.

In the approximation that the D2-brane is a test object, F_2 is independent of $x_1, \ldots, x_6$. Thus, we may begin by integrating $*_s \tilde{F}_4$ over each surface $x_9 = x_9^0, \rho = \rho_0, \psi = \psi_0$. The result must be one-half the charge of the D2-brane times a sign, depending on whether this surface passes above or below the D2-brane.[3] As a result, the diffuse charge in V_8 is 1/2 of the flux F_2 captured by V_8 above the D2-brane minus 1/2 of the flux captured below the D2-brane. In particular, for $\psi^{\min} = 0, \psi^{\max} = 2\pi$, we have

$$Q^{\text{Maxwell}}_{F_1,V_8} = \int_{V_8} *_s j^{\text{Maxwell}}_{D2} = \frac{T_{F1}}{2}\left(1 - \cos\theta_0 \quad \begin{matrix} +F_2 \text{ flux above brane} \\ -F_2 \text{ flux below brane} \end{matrix}\right), \tag{2.16}$$

where θ_0 is the value of θ at which V_8 intersects the D2-brane.

Note that, as mentioned in [32] for the similar D4/D6 case, the Maxwell charge remains diffuse for large ρ_0; i.e., 'near infinity.' Even when V_8 is contractible, $Q^{\text{Maxwell}}_{F1,V_8}$ need not vanish and, by an appropriate choice of V_8, $Q^{\text{Maxwell}}_{F1,V_8}$ can be made to take on a continuum of values. Perhaps the most interesting case is when V_8 is taken to be the entire cylinder (or, equivalently, the entire sphere) at infinity. This might be said to give the total Maxwell string charge created in the spacetime and flowing to infinity. Since $\theta_0 \to \pi/2$ for large ρ_0, the F_2 fluxes above and below the brane cancel and the total Maxwell charge flowing to infinity is just $T_{F1}/2$. In this case, Maxwell charge does not shed any particular light on charge quantization. The difference between the present setting and that of [30] can be traced to the presence of diffuse charge at infinity.

Finally, let us consider the Page current defined by

$$*_s j^{\text{Page}}_{F1} = d(e^{-2\phi} *_s H_3 - A_1 \wedge *_s \tilde{F}_4) = *_s j^{bs}_{F1} - A_1 \wedge *_s j^{bs}_{D2}\,. \tag{2.17}$$

Although this current is gauge dependent, the corresponding charge is naturally quantized. Integrating (2.17) over some V_8 yields a charge which is invariant under small diffeomorphisms, though it transforms under large diffeomorphisms. A study of the Kaluza-Klein reduction from 11-dimensions shows that the F1-brane Page charge associated with some region V_8 is identical to the M2-brane charge in the 8-volume given by lifting V_8 to the 11-dimensional spacetime using the supplementary condition $x_{10} =$ constant. The change of the Page charge under a large diffeomorphism $A_1 \to A_1 - d\Lambda$ just corresponds to the fact that the x_{10} coordinate transforms as $x_{10} \to x_{10} - \Lambda$, so that V_8 may now lift to a different surface in 11-dimensions. Since the Page charge is quantized, it should not lead to the discovery of any half-branes.

[3] In the present setting without D4 or NS5-branes, the three notions of D2-brane charge from [32] all agree.

Let us now work out the Page charge for our solution. It is clear from (2.17) that Page charge will be localized on the brane. When dealing with the Page charge, it is important to avoid integrating through a Dirac string. We therefore choose the gauge $A_1 = \frac{1}{2}(1-\cos\theta)d\psi$ with the Dirac string along the positive x_9 axis. Consider then the surface V_8 described just after eq. (2.15) for $\psi^{\min} = 0$, $\psi^{\max} = 2\pi$. If this surface is not to intersect the D6-brane, we should have $x_9^{\min}$, $x_9^{\max} < 0$. In this case, V_8 has two boundaries $\Sigma_{\min}$, $\Sigma_{\max}$ at $\rho = \rho_0$, $x_9 = x_9^{\max}$, $x_9^{\min}$. Thus, we have

$$\int_{V_8} *_s j_{F1}^{\text{Page}} = \left(\int_{\Sigma^{\max}} - \int_{\Sigma^{\min}} \right) (e^{-2\phi} *_s H_3 + *_s \tilde{F}_4 \wedge A_1) \,. \tag{2.18}$$

Now, the supergravity equations of motion state that

$$d(e^{-2\phi} *_s H_3 - *_s \tilde{F}_4 \wedge A_1) = 0 \,, \tag{2.19}$$

when the brane-source charge vanishes. Thus, the integrals over $\Sigma^{\min}, \Sigma^{\max}$ are unchanged by a homotopy that does not move them through any branes or Dirac strings. But, in our case, both surfaces are contractible. Thus, both integrals vanish and the Page charge is zero.

It is also interesting to compute the Page charge in other gauges of the form $A_1 = \frac{1}{2}(1 + 2n - \cos\theta)d\psi$ for integer n. Under a change of gauge, we see that the Page current changes by $\Delta j_{F1}^{\text{Page}} = -\Delta A_1 \wedge *_s j_{D2}^{bs}$. Thus, in the gauge labelled by n, there are n units of fundamental string Page charge flowing outward along the brane to infinity. As expected, the Page charge is quantized. One can think of this charge as entering the D2-brane along the Dirac string. From the above discussion of Kaluza-Klein reduction, it is clear that it is the Page charge that appears in the supersymmetry algebra and indeed this is the standard result (see, e.g., [41]). It is therefore surprising that one can find fundamental string charge radiating to infinity in what, by construction, is a solution with Killing spinors. Certainly, in the absence of the 6-brane, a bundle of fundamental strings that fan radially outward would be sure to break supersymmetry. It seems that the complicated asymptotic structure of our solution makes supersymmetry subtle in gauges where the D2-brane is topologically non-trivial due to its intersection with the Dirac string.

In the present setting, we have seen that the issue of half-branes is really a question of proper accounting. The constraint requires that the D2-brane be the source of 1/2 unit of fundamental string brane source charge. This is not a problem as in general it is the Page charge that should be quantized and not the brane source charge. Indeed, we have seen that the Page charge takes integer values.

However, in the above example a half unit of brane source charge does flow to infinity along a nearly flat D2-brane. Note that since brane source charge is a physical (i.e., gauge invariant) notion, such a brane source current will prevent compactification of the D2-brane, despite the vanishing of the Page charge in the simplest gauge. The issue is similar to the familiar fact that the total electric charge must vanish on a compact space. Thus, the example considered here is not related by T-duality to the D0/D8 case.

In order to be compactified, a mostly flat D2-brane must have an induced F1 brane-source current that vanishes at infinity. However, considering a spacetime of the form $\mathbb{R}^8 \times T^2$ in which a D2-brane wraps some T^2 transverse to a D6-brane, we see that such a D2-brane must still be the source of a net 1/2 unit of string brane-source charge. Thus, the D2-brane must deform so that either 1) a part of it represents a string running off to infinity in some remaining non-compact direction or so that 2) it intersects the D6-brane, into which the F1-brane charge can flow. In either case, mere accounting issues cannot rid us of the factor of 1/2. To see this, we remind the reader that the brane source and Page currents for fundamental strings differ only by a term of the form $A_1 \wedge *_s j^{bs}_{D2}$. In a region where the D2-brane takes the shape of a long narrow tube (i.e., a string), we can arrange to move the Dirac string elsewhere and to have A_1 smooth near the string. If the tube is thin,[4] integrating A_1 around the tube must then give zero. Thus, under such circumstances, the brane source and Page charges of the string will be equal. If the D2- and D6-branes meet and intersect, we can create such a thin tube by moving the flat part of the D2-brane far from the D6-brane (in analogy with figure 2).

We conclude that an understanding of cases in which the worldvolume of the D2-brane is compact requires an analysis of the actual intersection of the D2- and D6-branes. Due to the D6-brane singularity, it is not clear from the type-IIA perspective which intersections are in fact allowed. Since, however, the core of a unit charged Kaluza-Klein monopole is smooth, the question is much more tractable in the 11-dimensional formulation. It is to this question that we turn in the next section.

3. Intersecting D2- and D6-branes

We have seen that, starting with a smooth holomorphic curve in the Kaluza-Klein monopole spacetime, dimensional reduction can yield a type-IIA configuration in which 1/2 unit of fundamental string charge exits to infinity. In this case, the fundamental string charge is dissolved within a D2-brane. One might wonder whether one can in fact find holomorphic curves whose reduction to ten dimensions yields 1/2 strings in more familiar string-like configurations which are not dissolved inside the D2-branes. Here, one may take inspiration from [9], where a number of solutions were found to the Born-Infeld equations of motion for a D5-brane in the background of a D3-brane which yield string-like configurations when parameters are properly adjusted. Similar configurations for D3-branes in a background of NS5-branes were recently constructed in [35].

[4] One might also consider configurations in which a finite width cylinder of D2-brane extends to infinity. Non-BPS initial data of this sort can be given in certain cases, but energy arguments imply that this cylinder should contract.

The solutions studied in [9, 35] were rotationally symmetric and BPS.[5] When the test D-brane was moved far past the background brane, the world volume of the test brane is deformed such that it forms a string stretching between the main part of the test brane and the background brane. The tension of this string was computed in [9] and was found to depend on the total amount of background flux that had been captured by the test-brane. Although the context is slightly different, this is in direct parallel with the constraint (2.13) which describes the creation of strings. Solutions that appeared to contain fractional strings arose when the test D-brane intersected the background brane. The configuration is illustrated in the diagram below. The test brane formed a cone near the background brane, and the tension of the string was related to the opening angle of this cone. In the symmetric case where the opening angle is $\pi/2$, the tension was found to vanish. In [35], it was argued that a corresponding opening angle was quantized for consistency with charge quantization.

The authors of [9] question the physical relevance of such solutions, as the Born-Infeld description of the test-brane cannot describe the details of the intersection between the test-brane and the background brane. It is not clear that this should have been a concern, as the background generated by a highly charged D3-brane is a large black brane with a smooth horizon, which is well described by supergravity. In any case, the situation here is well under control. When lifted to M-theory, the core of the D6-brane becomes just the center of a unit charged Kaluza-Klein monopole, which is perfectly smooth. As a result, precisely those solutions which correspond to a smooth M2 test-brane should reduce to a valid test brane configurations in type IIA.

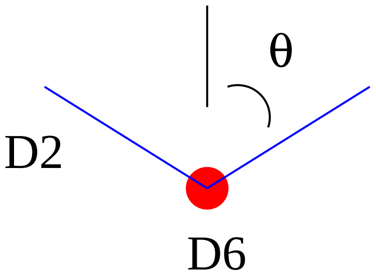

Figure 4: The D2-brane takes the shape of a cone with opening angle θ and vertex at the D6-brane.

In order to find such a solution, let us note that the simplest test-brane configuration with opening angle $\pi/2$ is simply given by placing the D2-brane on the surface $x_9 = 0$. A holomorphic curve that reduces to this surface can be found by recalling that the symmetry $(w, v) \to (v/w, v)$ changes the sign of x_9, so that any surface which is invariant under this symmetry must lie at $x_9 = 0$. One can readily verify that the surface $w^2 = v$ is in fact invariant. However, because it contains w^2, upon dimensional reduction we find *two* D2-branes lying at $x_9 = 0$. The corresponding sheets of the M2-brane lie at $x_{10} = \psi$ and $x_{10} = \psi + \pi$. Deforming this surface to move the asymptotic parts of the D2 branes to large $|x_9|$ therefore produces a string-like piece of D2-brane carrying a full unit of fundamental string charge. Note that the two D2-branes cannot be separated from one another as, when the angle

[5]The authors of [9] argued that their solutions were likely to be BPS, while the solutions of [35] are conclusively so.

ψ increases by 2π, we must move from one brane to the other. Note also that a $\mathbb{Z}_2$ quotient of this space leads to the charge 2 Kaluza-Klein monopole and the charge 2 D6-brane. Such a $\mathbb{Z}_2$ quotient identifies the two sheets of the M2-brane, leading to a single D2-brane in the charge 2 D6-brane background.

For completeness, let us consider the most general holomorphic curves that lead to rotationally symmetric configurations in type-IIA theory. That is to say, we wish to consider curves such that if the curve intersects m times with the fiber over (v, x_9), then it also intersects m times with the fiber over $(e^{i\psi_0}v, x_9)$. This implies that the holomorphic curve is of the form $v^n = e^b w^m$. Since v, w are smooth coordinates away from the positive x_9 axis (i.e., $\rho = 0$, $x_9 \geq 0$), any $v^n = e^b w^m$ that does not intersect this axis is in fact smooth. Similarly, the $\mathbb{Z}_2$ symmetry tells us that any such curve which avoids the negative x_9 axis ($\rho = 0, x_9 \leq 0$) is smooth. As a result, we may conclude that all curves $v^n = e^b w^m$ which do not intersect the origin $\rho = x_9 = 0$ are smooth.

To study the curves near the origin, we note that $x_0, x_1, \ldots, x_6$ together with the complex coordinates $\beta = w, \alpha = v/w$ form a smooth coordinate system near $x_7 = x_8 = x_9 = 0$. One may check that near this point the metric takes the form $ds^2 = d\alpha d\overline{\alpha} + d\beta d\overline{\beta} + O(\alpha) + O(\beta)$. Note that the $\mathbb{Z}_2$ symmetry merely interchanges α and β. It is useful to note here that our α is the complex coordinate called v in [7, 6] while our β is the complex coordinate called $\overline{w}$ there. Since the curves may be written $\alpha^n = e^b \beta^{m-n}$, it is clear that curves intersecting the origin are smooth for $n = 1$ (and $m - n > 0$) or $n = m - 1 > 0$, but not for other cases. The $\mathbb{Z}_2$ symmetry interchanges $n = 1$ with $n = m - 1$.

What is interesting about all of this is that, from the point of view of the IIA spacetime, the curves do not appear to intersect the D6-brane in a smooth way. The curves satisfy

$$x_9 = \frac{1}{2} e^{\frac{b+\overline{b}}{m}} e^{-2x_9} \rho^{2(1-n/m)} - \frac{1}{2} e^{-\frac{b+\overline{b}}{m}} e^{+2x_9} \rho^{2n/m}, \tag{3.1}$$

so that near the origin they will in general have the form of a cusp. Let us take the case $n = 1$. Such curves intersect the origin smoothly only for $m \geq 2$, in which case we find:

$$x_9 \sim \rho^{2n/m}. \tag{3.2}$$

For the special case $n = 1$, $m = 2$ which we argued might give rise to half-branes, the curve is a cone near the origin, satisfying

$$x_9 \sim \rho \sinh\left(\frac{b + \overline{b}}{2}\right). \tag{3.3}$$

Since these results correspond to smooth holomorphic curves in the eleven dimensional space, we conclude that they do in fact describe BPS configurations of D2 branes intersecting a D6-brane.

It is interesting to compare our cone-shaped solutions ($n = 1, m = 2$) to those of [9]. Our solutions are more restrictive, as the asymptotic (large ρ) location of our D2-branes is correlated with the opening angle of the cone. In particular, for large ρ our $n = 1, m = 2$ solutions approach[6] $x_9 = \frac{b+\bar{b}}{4}$. In contrast, the opening angle and asymptotic location were separate parameters in [9]. This might possibly be due to the lower (co-)dimensional nature of our system. For fundamental strings attached to a larger brane, the brane should flatten more quickly at large ρ which might allow the asymptotic x_9 position and the opening angle to be tuned separately.

We should, of course, evaluate the flux of fundamental strings carried by the general curve, $w^m = e^{-b}v^n$. The calculation proceeds much as in section 2. The result is that the flux of strings passing through a surface $\rho = constant$ or $x_9 = constant$ is given by:

$$\text{Brane-source F1 Flux} = \frac{1}{2}[(m - 2n) - m\cos\theta], \tag{3.4}$$

in units of fundamental string charge quanta. At large ρ, one may check that the surface satisfies $x_9 = \pm|1/2 - n/m| \ln\rho$ so that, in particular, $\theta \to \pi/2$ as $\rho \to \infty$. Thus, it follows that such configurations have net flux of $\frac{m-2n}{2}$ strings running out to infinity (and not those flowing into the D6-brane) along m D2-branes. We note that it is the number of strings exiting to infinity that controls the asymptotic shape of the D2-brane. This observation may be important for fixing the proper relationship between the tension computations of [9] and charge quantization. The coefficient of $\ln\rho$ is just what one would find for m D2-branes attached to $\frac{m-2n}{2}$ fundamental strings.

These configurations contain m D2-branes, so that the term $\frac{m}{2}\cos\theta$ is consistent with our constraints determining the rate of creation of string charge. Note that in regions where $\theta = 0$ and $\cos\theta = 1$, the total flux of strings is always an integer (n). Due to the symmetries of our setting, long thin tubes approximating strings can arise only at such values of θ. In particular, for $n = 1, m > 2$, the tube always becomes thin at the cusp where it intersects the D6-brane. Thus, an integer amount of Page charge flows into the D6-brane and the D2- and D6-branes may be said to be connected by an integer number of branes.

The most interesting case is $n = 1, m = 2$ for which the brane source charge vanishes at infinity. Note that this case maps to itself (with $b \to -b$) under the $\mathbb{Z}_2$ reflection. It is only this case that one might hope to compactify and relate to the D0/D8 system via T-duality. As we have seen, the corresponding D2-brane intersects the D6-brane and does not run off to infinity. In the usual way, the Page current is conserved and is related to the brane-source current by

$$*_s j_{F1}^{\text{Page}} = *_s j_{F1}^{bs} - A_1 \wedge *_s j_{D2}^{bs}\,. \tag{3.5}$$

[6]As may be verified from (3.1), the usual logarithmic divergence associated with a string intersecting a D2-brane is absent in this case.

Conservation means that it suffices to compute the charge at infinity. The case $n = 1, m = 2$ contains two D2-branes, we see that in the simplest gauge $A_1 = \frac{1}{2}(1 - \cos\theta)$ we have one unit of Page charge on the brane. In particular, the string connecting the D2- and D6-branes has one unit of Page charge. In the more general gauge $A_1 = \frac{1}{2}(1 + 2n - \cos\theta)$, we have $(2n + 1)$ units of Page charge. It appears that, for this case, there is no gauge in which the Page charge vanishes. Similar comments hold for the Page charge in the other smooth cases. In all cases, diffuse Maxwell charge is present at infinity.

4. Discussion

In the above work, we have studied smooth holomorphic curves representing test M2-branes in the background of a Kaluza-Klein monopole. The associated distributions of charge in type-IIA theory have been computed for all cases where the reduction to type IIA is rotationally symmetric. This has allowed us to construct a number of solutions where the intersection of the D2-brane and the D6-brane appears singular from the type-IIA perspective, although the 11-dimensional perspective reveals that it is in fact smooth and BPS. As a result, we expect that all of the solutions[7] of [9] do in fact correspond to valid BPS solutions.

Perhaps the most interesting part of this work is that it provides insight into issues involving half-strings that arise in the context of the Hanany-Witten effect. When a single D2-brane is stretched above a D6-brane as in figure 1 the constraint (2.13) requires it to be the source of 1/2 unit of brane-source charge. We have seen that in certain cases where this half-string runs to infinity along a flat D2-brane, the half-string is merely a matter of accounting. In general it is the Page charge, as opposed to the brane source charge, that is quantized and the Page charges for such cases turn out to be integral.

However, such a string charge running to infinity along a mostly flat D2-brane will prevent the brane from being compactified. The effect is much the same as the familiar statement that a compact space must contain zero total charge.

In the Hanany-Witten setting, one typically assumes that the brane source charge leaves the D2-brane by flowing into the larger D6-brane. In this case, the D2-brane and D6-brane must intersect. If we now imagine pulling the main body of the D2-brane far away from the D6-brane, they would remain connected by a thin tube that approximates a fundamental string. However, we have seen that the brane-source and Page charges of such a thin tube agree, so that mere accounting cannot in this case explain the factor of 1/2.

It is therefore natural to suspect, in the case where the x_7 and x_8 directions are compactified and the D2-brane is compact, that only such combinations of branes

[7] Or, at least, that dense set for which the string charges can be made integral by considering some integer number of copies.

can arise for which half-strings are not needed. In section 3, we found some evidence that M-theory provides a mechanism to enforce this constraint. Considering now the uncompactified case, we succeeded in finding smooth BPS M2-branes in the 11-dimensional Kaluza-Klein monopole for which a (single) corresponding D2-brane would be attached to the D6-brane by 1/2 of a fundamental string. Furthermore, the F1 brane-source charge for such D2-branes vanished at infinity so that no obstacle to compactification is expected. However, in order for our 11-dimensional surfaces to be smooth, they necessarily projected to a *pair* of D2-branes which were in fact joined to the D6-brane by a single unit-charged fundamental string. Thus, our results suggest that in a compactified background which contains a torus (T^2) transverse to the D6-brane, compact D2-branes wrapping this torus can arise only in pairs. In contrast, we were able to construct isolated D2-branes in the background of the charge 2 D6-brane – a case where again only integer charged strings are required.

The reader may rightly ask whether one can trust such classical arguments to tell us about fine points of charge quantization. It is clear that our description does have a regime of validity. When the x_{10} circle is large compared to the eleven-dimensional Planck scale, the Kaluza-Klein monopole description of the background and is very flat on the gravitational scale associated with an M2-brane. Thus, we expect that a collection of N M2-branes for $N \gg 1$ is well-described by a test brane in this background if the x_{10} circle is large enough. In this case it is clear that the corresponding IIA system has $2N$ units of D2-brane charge to leading order in N. However, if the actual D2-brane charge were $2N + 1$, so that the number of D2-branes was odd, then this would not resolve our dilemma about half-strings. While it is not clear how to rigorously rule out such $1/N$ corrections, the final picture is quite compelling. One might therefore say that our results constitute a strong plausibility argument rather than a proof. It is important to bear this in mind, but we will not consider such subtleties further here.

Let us take a moment to investigate this confinement mechanism more carefully by considering the D6-brane background with x_7 and x_8 compactified, and by dropping the restriction to the BPS cases. Equivalently, we may discuss periodic surfaces in a periodic two-dimensional array of unit charged D6-branes. Such a solution lifts to a multi-center Kaluza-Klein monopole metric [43] in eleven dimensions. This metric is again smooth and near the 'core' of each monopole has just the same structure as the single monopole metric. We are now interested in the question of classifying periodic smooth 2-surfaces in this background. The constraint (2.13) guarantees that, if the surface does not extend to infinity, it must intersect each of the Kaluza-Klein monopole cores.

Now, near any such intersection the surface must be approximately described by some plane $a_1\alpha + a_2\overline{\alpha} + b_1\beta + b_2\overline{\beta}$, where α, β are smooth complex coordinates near a given monopole core. An orbit of $\lambda = \partial_{x_{10}}$ intersects such a plane twice. Thus we find an even number of sheets of D2-branes. The sheets in fact form a Riemann surface, as traveling in a circle about the D6-brane moves us from one sheet to the next. Thus, pairs of sheets cannot be globally separated, though we can always introduce a local deformation of one sheet to separate it from the other in some small region of the x_7, x_8-plane. It is useful to note that an explicit solution which is the analogue of $v/w^2 = 1$ in the asymptotically flat case is easy to construct: this is again just the plane which is invariant under a $\mathbb{Z}_2$ symmetry which amounts to a reflection about the plane containing the D6-branes.

We now comment briefly on a potential exception to our $\mathbb{Z}_2$ confinement phenomenon. Consider the case where two of the directions transverse to the D6-brane are compactified on a T^2, but the third transverse direction remains non-compact. The potential loophole would involve a single D2-brane in the compactified system that does not intersect the D6-brane at all, but instead extends to infinity in the remaining non-compact direction and carries away some amount of fundamental string brane-source charge. Such configurations cannot be BPS or even static, as without a (half) fundamental string to hold them together, the D6- and D2-brane repel each other [16] and the D2-brane will retreat to infinity. Nonetheless, initial data of this sort appears to exist for the D2-brane Born-Infeld system and it seems to lift smoothly to eleven dimensions. It would be interesting to understand that status of such configurations in detail but, for the moment, we set aside this possible exception.

Let us now ask what our results have to say about the D0/D8 system. Intuitively, we expect these systems to be related by T-duality. Unfortunately, it is difficult to apply T-duality precisely in this case. The curved branes make it difficult to apply T-duality in a controlled way in the perturbative setting [44], and similarly to apply Buscher T-duality in supergravity [45] we would have to 'smear' the system to create a Killing symmetry.

Nevertheless, we expect the D0/D8 system to behave analogously. This would mean that the symmetric unit charged D8 brane background, where the Ramond-Ramond field strength takes the values $\pm 1/2$ of the fundamental quantum on either side, is allowed. However, in such a background D0-branes would occur only in pairs. There is clearly no analogue of the 'local separation' that was allowed for the compact D2-branes.

An interesting question is what gauge group would arise on the pairs of D0-branes. We recall that the type-I fivebrane also arises only in pairs, at least as counted in a sense natural to the type-IIB description. There, the gauge group on such a pair is SU(2) [38, 39]. In the case of the pair of D0-branes in the background of a single D8-brane, one again expects the group to be some projection of U(2); i.e., either SU(2) or U(1) depending on whether the projection is symmetric or antisymmetric.

Now, we do not normally expect a semiclassical description of the sort used here to tell us about the detailed structure of the gauge groups. Also, we realize that in the D2-brane case the branes are allowed to separate locally and are 'confined' only in some global sense. Thus, we expect that the gauge group in that case is locally U(2) but with some global constraint. To get some idea about this global constraint, let us consider the case where the two sheets of D2-brane in fact coincide. Then, we can make two observations that suggest that the gauge group in the present setting should include global U(1) rotations instead of global SU(2) rotations. First, we recall from section 3 that, although the two D2-branes coincide the corresponding sheets of the M2-brane are separated in x_{10}. This is clearly true for the $(n = 1, m = 2)$ case $v/w^2 = e^b$. Since the two sheets of the M2-brane do not coincide, we would be surprised to find a group that is not a subgroup of $\mathrm{U}(1) \times \mathrm{U}(1)$. Second, although T-duality cannot be used in a rigorous way, we expect T-duality to map our pair of D2-branes to a pair of D0-branes that are stuck together, and in particular to a pair of D0-branes located at the same point in space. Thus, one expects that holonomies of the gauge fields on the D2-branes around the non-trivial cycles of the torus take values of the form $\begin{bmatrix} e^{i\phi} & 0 \\ 0 & e^{i\phi} \end{bmatrix}$, which do not arise in SU(2). This is in contrast with the case of type-I fivebranes, for which an M-theoretic explanation of the antisymmetric projection condition leading to SU(2) can be related to two branes having mirror-symmetric positions across a would-be orientifold plane.

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